Calculating Main Sequence Lifetimes

At the beginning of the twentieth century two astronomers, the Danish E. Hertzsprung and the American H. N. Russell, established a correlation between two important stellar parameters: brightness and color.

Since ancient times, the brightness of a star is indicated by "magnitudes": 1, 2 and so on, with dim stars having larger magnitudes. Don't confuse the relative magnitude with absolute magnitude. The *relative magnitude* measures the brightness of a star as it appears in the sky and it depends on the brightness and on the distance; if we put a star at the distance of 10 Parsec (33 year light), its magnitude is called *absolute magnitude*: obviously it depends only on brightness of the star.

According to color, stars are grouped in classes: O, B, A, F, G, K, and M. O stars are blue and M are red - see Table A. Classes are oddly sequenced because they were assigned long before we understood their relationship to surface temperature: O stars are the hottest and M the coolest ones; O and B stars are rare but very bright; M stars are numerous but dim.

Hertzsprung and Russell discovered that in a Color-Magnitude diagram (H-R diagram) stars appear distributed in well-defined groups, so that there is a correlation between temperature and luminosity (Fig. 1).

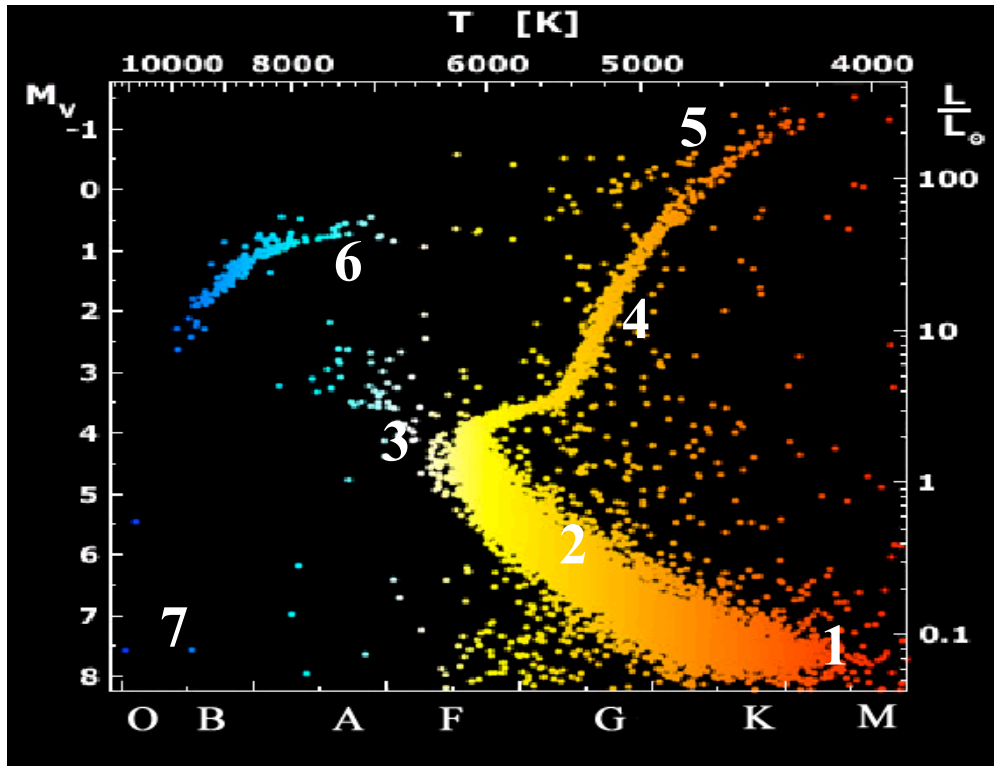| Spectral Class	Temperature (°K)	Color	Example
O	>25000	Blue	ξ Puppis
B	25000-11000	Blue	β Orionis (Rigel)
A	11000-7500	Blueish White	α Canis Majoris (Sirio) - α Lyrae (Vega)
F	7500-6000	White	α Canis Minors (Procione)
G	6000-5000	Yello...	α Aurigae (Capella) − Sun
K	5000-3500	Oran...	α Bootis − α Tauri (Aldebaran)
M	3500-2200	Red	α Orionis (Betelgeuse) − α Scorpii (Antares)

Table A

Fig. 1

The H-R diagram provides a full interpretation in terms of stellar evolution theory. With reference to fig. 1, a short description of stellar evolution says:

1. The star is just born: its temperature rises, but not its luminosity. During this phase, thermonuclear reactions involve the burning of Hydrogen.
2. The star is into main sequence: this is a period of stability in which the star burns the Hydrogen producing Helium. The star spends most of its time in the main sequence; we can calculate how long the star stays on main sequence, using this easy equation (m=mass not magnitude):

$$t_{m.s.} = 10^{10} \times \left(\frac{m_{Sun}}{m_{star}} \right)^3 Years \qquad [1.1]$$

If we remember that the Sun's mass is 2×10^{30} Kg, and knowing the stellar mass, we can calculate the period in which the star burns Hydrogen, which the time it stays as a main sequence star.

3. The star *turns off*: the star exhausts the Hydrogen in its core and it continues to burn it in shells.. For a 'short' period the stellar temperature decreases without a great change of luminosity.
4. The star burns Hydrogen in shell (red giants) through the CNO cycle: as its radius increases so its luminosity raises.
5. *Helium flash*: the star begins to burn Helium maintaining the combustion of Hydrogen in shells. The stellar temperature increases very quickly, because the new reactions give off a great quantity of energy.
6. The star goes through a fast period of luminosity variability (called the *Horizontal branch*) till it reaches a new configuration of equilibrium during which the star burns Helium in its core and Hydrogen in its shell.
7. If the stellar mass is not enough to light new thermonuclear reaction beyond the Helium burning, it 'dies' as a white dwarfs of Helium.

In Fig. 1, there no stars with a mass greater than 7-8 solar masses. The stars with these masses are able to begin new fusion reactions involving the burning of Helium, Carbon, Oxygen, Magnesium and Neon. A star with a mass greater than 10 solar masses can develop thermonuclear reactions until it creates Iron. This will cause an ending to its life in a supernova type II.

The *luminosity (L)* of a star is the energy radiated from all of its surface in one second. The *absolute magnitude* (*M*) is used usually to measure the luminosity of the stars; it is related to the stellar luminosity by an approximate relation:

$$M \cong -2.5\log\left(\frac{L_{star}}{L_{Sun}}\right) + 5$$

To drop the log, we can use the inverse formula:

$$\frac{L_{star}}{L_{Sun}} \cong 100 \times 10^{\left(-\frac{2}{5}M\right)}$$

[1.2]

We remember that Sun's luminosity is: $L_{Sun} = 4 \times 10^{26}$ Watt. The stellar luminosity is connected by the mass through this correlation:

$$L_{Star} : L_{Sun} = m_{Star}^{3.5} : m_{Sun}^{3.5}$$

[1.3]

Using all these easy relationships and the HR diagram, if we know one parameter, we are able to estimate the rest. As is common in Astronomy, the star's parameters are in comparison to the Sun.

For example, the *absolute magnitude* of Sirius (α Canis Majoris) is *M=+1.5*. With this parameter, using [1.2] we can find $L_{Star}\big/L_{Sun}$ = 25 ; [L_{star}/L_{Sun}=100(10^(-2/5x1.5) => L_{star}/L_{Sun}=25 or L_{star}=25L_{Sun}].

Then using [1.3] we obtain its mass: $m_{\alpha CM} = 2.5 \times m_{Sun}$;
[25=$m_{star}^{3.5}$/$m_{Sun}^{3.5}$ => (m_{star}/m_{Sun})$^{3.5}$=25 => m_{star}=2.5m_{Sun}].

Using your results from equation 1.2 and the HR diagram, ass illustrated at right, we can estimate the spectral class of Sirius (A).

Lastly, to calculate how long the star stays in the main sequence use the equation [1.1] and the value is 0.64 billion years.
[from eq 1.3 m_{star}=25m_{Sun} then use eq 1.1
t=(10^10)*(m_{Sun}/m_{star})^3
=> t=(10^10)*(m_{Sun}/2.5m_{Sun})^3=(10^10)*(1/2.5)^3=6.4*10^8].
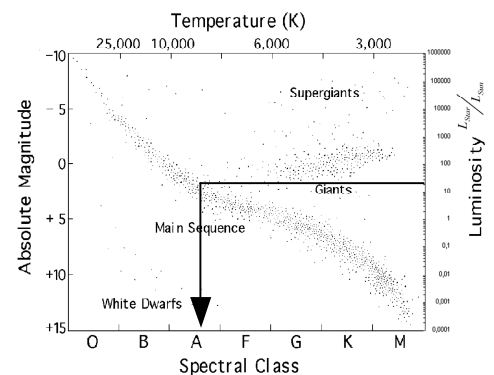
In conclusion, if we have the absolute magnitude of a star, connecting all the equations, we are able to estimate these parameters: luminosity, mass, spectral class (then temperature) and the time the star is in the main sequence stage.

Look over the three formulas listed below, the HR diagram below and the attached Table 1, which lists the absolute magnitude of some important and familiar stars. Use this information to complete Table 1. Then answer the questions below on the back of Table 1.

Turn in Table 1 and the answers to the following questions

- What kind of relationship is there between the absolute magnitude and luminosity?
- What kind of relationship is there between the luminosity and the stellar mass?
- What kind of relationship is there between the mass and the time spent in the main sequence?

$$\frac{L_{star}}{L_{Sun}} = 100 \times 10^{\left(-\frac{2}{5}M\right)}$$

$$t_{m.s.} = 10^{10} \times \left(\frac{m_{Sun}}{m_{star}}\right)^3 \; Years$$

$$L_{Star} : L_{Sun} = m_{Star}^{3.5} : m_{Sun}^{3.5}$$

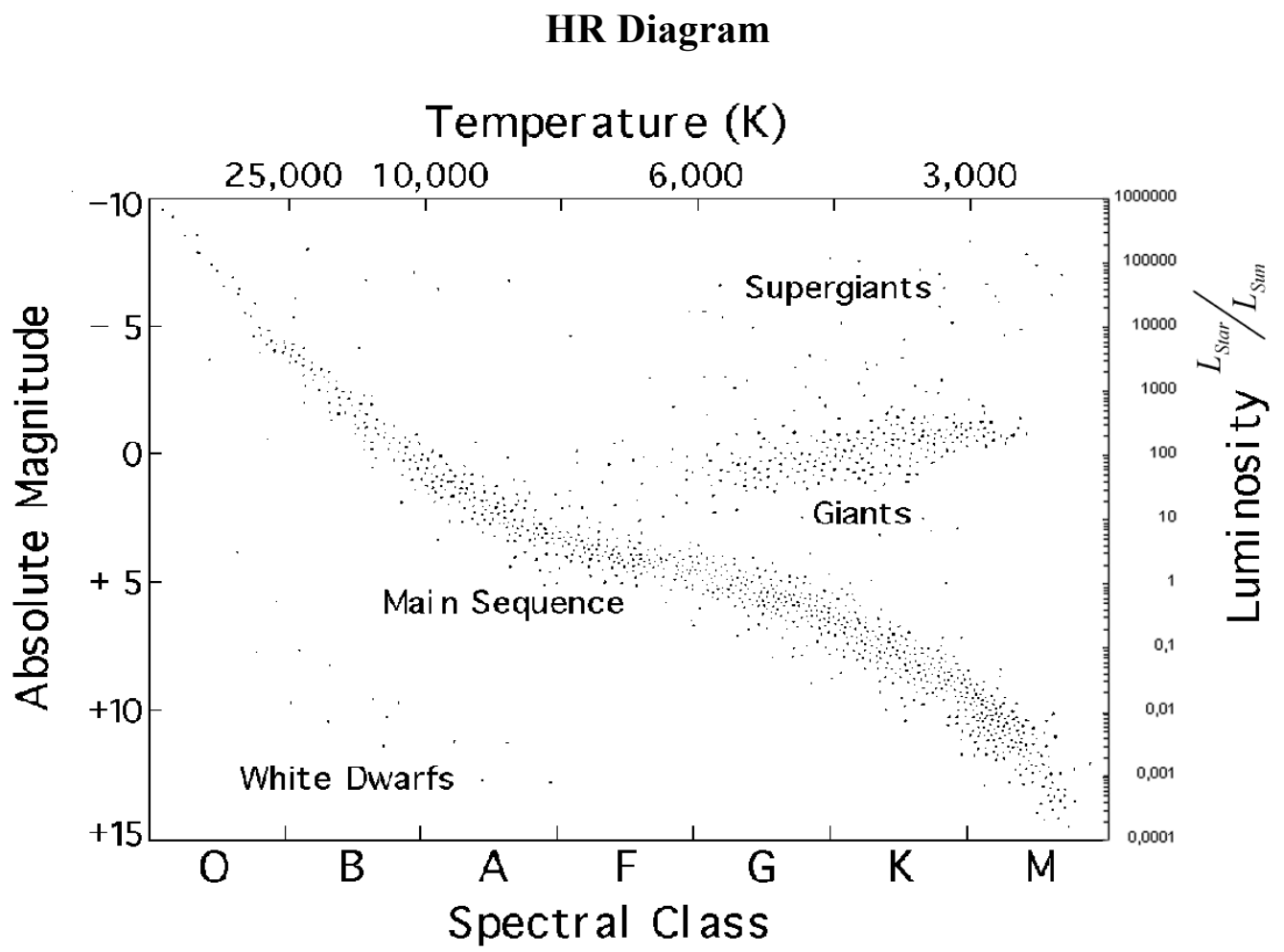## HR Diagram

Table 1

Star	Absolute Magnitude	$\frac{L_{star}}{L_{Sun}}$	Mass	Spectral class	Main sequence lifetime
Sirius (α Canis Majoris)	+1.5				
Arcturus (α Bootis)	-0.3				
Vega(α Lyrae)	+0.5				
Capella (α Aurigae)	-0.6				
Rigel (β Orionis)	-8.2				
Procyon (α Canis Minoris)	+2.7				
Altair (α Aquilae)	+2.4				
Aldebaran (α Tauri)	-0.6				
Antares (α Scorpii)	-5.0				
Spica (α Virginis)	-2.9				
Polaris (α Ursae Minoris)	-3.7				
Deneb (α Cygni)	-6.2				
Regulus (α Leonis)	-0.2				